Bolstering the Modelling and Forecasting of Realized Covariance Matrices using (Directional) Common Jumps

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Abstract

This paper proposes a robust framework for disentangling undiversifiable common jumps within the realized covariance matrix. Simultaneous jumps detected in our empirical study are strongly related to major financial and economic news, and their occurrence raises correlation and persistence among assets. Our application to 20 Dow Jones stocks, shows that common jumps and directional common jumps substantially improve the in- and out-of-sample forecasts of the realized covariances at the day-, week- and month-horizon. Applying these new specifications to minimum variance portfolios results in superior positions from reduced turnover. The implication is that investors willingly sacrifice up to 100 annual basis points in switching to those strategies.

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JEL codes: C14; C32, C58, G11, G32.

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1 Introduction

Market crashes and sudden reactions to major financial news generally trigger the occurrence of common jumps in several stocks, thereby raising statistical correlations among asset prices in consequence of enhanced market-wide information. As this phenomenon raises short-term predictability, it increases (decreases) the persistence of covariances when the common jumps are associated with bad (good) news. The high levels of correlation among common jumps, and the changes in the persistence of covariances, shed light on their rich information content for modelling and forecasting realized covariance matrices.

This paper proposes a robust non-parametric framework for measuring separately the common jumps and continuous components of the quadratic covariation matrix. Our approach builds directly on the theoretical results of Barndorff-Nielsen and Shephard (2004a) and Mancini and Gobbi (2012) that involve the use of so-called realized and truncated realized covariation. The divergence between these two estimators leads to a matrix of common jumps, as identified with the multi-jump test of Caporin et al. (2017). By employing the approach of the latter, we are able to detect days where all elements in the common jump matrix are distinctly different from zero. We then demonstrate significant forecasting and economic gains at the daily, weekly and monthly horizons, as attained by models that utilize the information of common jumps. Furthermore, we construct measures of directional common jumps, and investigate whether the sign of 'news' contains further explanatory power. Metrics are estimated as the difference between the positive and negative realized semicovariances (e.g. Bollerslev et al., 2020).

Common jumps have many implications for portfolio allocation, risk management, and forecasting. As noted by Das and Uppal (2004) and Longin and Solnik (2001) the increased correlation, that is associated with a general market crash, reduces the diversification potential of portfolio and risk managers. Common jumps are also likely to affect the aggregate attitude to risk, with obvious effects upon risk premia. For instance, Bollerslev and Todorov (2011); Bollerslev et al. (2015) show that the risk compensation for large jumps is also large and time-varying. Separating the impact of the continuous and common jumps components is also crucial for forecasting covariances. The different explanatory factors for these distinct sources of risk have to be considered with different coefficients in order to account for the Brownian correlations and common jumps (e.g. Andersen et al., 2007a; Corsi et al., 2010, for a similar rationale in a univariate framework).

Contrasting with the volatility forecasting literature, where the role of jumps has been extensively studied (e.g. Andersen et al., 2007a; Busch et al., 2011; Corsi et al., 2010; Duong and Swanson, 2015; Patton and Sheppard, 2015, inter alios), the literature on covariance forecasting largely ignores common jumps. To the best of our knowledge, only Asai et al. (2020) have considered the relevance of common jumps in forecasting bi-variate volatility. Yet, for asset allocations, it is vital to understand the role of common jumps for a large set of assets, as the effect of common jumps in a pair of assets is negligible in large portfolios. A related study (Caporin et al., 2017) that focuses on univariate volatility, finds common jumps to have a greater impact upon volatility than univariate jumps. The procedure of that study allows the detection of simultaneous jumps for a large number of assets. It thereby demonstrates the multi-jump test to be more powerful than the co-exceedance rule (e.g. Gilder et al., 2014),[1] which fails to identify common jumps that are realized with small lags, and which suffer from both, the slow incorporation of news of low volume stocks and the volatility spikes that are generally associated with common jumps.[2]

Our empirical application considers 20 individual Dow Jones stocks for the period 2000–2016. A preview of our results is as follows: we find common jumps to be strongly associated with major financial and economic news. In particular, we find that FOMC announcements generally trigger simultaneous common jumps, with jump sizes between 0.8–2.0%. Alternatively, flash crashes are associated with jump sizes between 1.5–5% (see Figure 1). These results are in line with the findings of Aït-Sahalia and Xiu (2016),

[1]The co-exceedance rule identifies common jumps by intersecting univariate intraday jump tests. For univariate intraday jump tests see Andersen et al. (2007b); Lee and Mykland (2008), while for daily jump tests see Andersen et al. (2012); Barndorff-Nielsen and Shephard (2004b, 2006); Corsi et al. (2010), among others.

[2]The test of Jacod and Todorov (2009), Mancini and Gobbi (2012) and Bibinger and Winkelmann (2015) also identify common jumps, but they are limited to a pair of assets. The test of Bollerslev et al. (2008) identifies common jumps using an aggregate market index.

Dungey and Hvozdyk (2012) and Lahaye et al. (2011), that macroeconomic announcements are sufficient to explain the occurrence of common jumps, as they significantly change the probability of observing common jumps.

In extending the vech-HAR model of Chiriac and Voev (2011) to account for common jumps, we propose the vech-HARJ and vech-HARCJ models, which are multivariate extensions of the HARJ and HARCJ of Andersen et al. (2007a).[3] Whereas the HARJ model augments the vech-HAR by incorporating a daily common jump variable, the HARCJ model uses the daily, weekly, and monthly levels of the continuous and common jump variables to model future covariance matrices. The incorporation of common jumps results in large in- and out-of-sample improvements vis-à-vis the HAR model; but the HARCJ model delivers larger forecasting gains across all horizons. In general, models based on directional common jumps, deliver forecasts that improve upon the in- and out-of-sample performance of the HAR model. However, their forecasts are inferior to that of models using common jumps.

To assess the relative economic value of the different models, we construct global minimum variance portfolios, which we evaluate using a utility-based approach, as in Fleming et al. (2001, 2003). The use of common and directional common jumps delivers statistical improvements, with economic gains arising from the enhanced accuracy associated with stable covariance matrices. Reduced turnover lowers trading costs: an investor with a risk-aversion of $\gamma = 6$, would be willing to sacrifice up to 100 annual basis points to switch to the models that utilize the common or directional common jumps.

Finally, using simulations of realistic price processes that accommodate for the presence of idiosyncratic and common jumps, we show that our framework successfully disentangles the continuous and common jump parts of the quadratic covariation, and their use in forecasting significantly outperform the forecasts of the standard multivariate HAR model.

The remainder of the paper is organized as follows: Section 2 presents the theoretical

[3] Multivariate GARCH models are popular alternatives available in the literature for modelling and forecasting covariances (e.g. Bollerslev, 1990; Engle, 2002; Engle and Kroner, 1995; Noureldin et al., 2012). However, the curse of dimensionality is of relevant consideration, as the number of parameters to be estimated grows very rapidly when the number of assets is large.

framework, where the multi-jump test and methodology employed for decomposing the covariance matrix into its continuous and common jumps parts are outlined. Multivariate models and the forecasting evaluation criteria are set out in Section 3. The Monte Carlo exercise is described in Section 4, and the simulated results are also presented. The data, occurrence of simultaneous jumps and their link with major financial and economic news are reported in Section 5. Here, we also report the in- and out-of-sample performance of the multivariate forecast models. Section 6 reports the incidence of directional common jumps in forecasting realized covariances. Section 7 presents the economic evaluation of the different models based on a utility-based approach. Section 8 concludes.

2 Theoretical Framework

Let $X = \left(X^{(i)}\right)_{i=1,\dots,N}$ be the log-prices of an N-dimensional vector of assets. We assume that stock prices evolve continuously on a filtered probability space $(\Omega, \mathcal{F}_t, \mathcal{F}_{t\geq 0}, \mathbb{P})$, and that dynamics for X are as follows:

$$X_t = x_0 + \int_0^t \mu_s ds + \int_0^t \Sigma_s dW_s + dJ_t \tag{1}$$

where μ_s is an N-dimensional drift term which is bounded and predictable, $\Sigma_s \equiv \sigma_s' \sigma_s$ is the instantaneous covariance, and W_s is an N-dimensional vector of independent Brownian motions. The unit time interval is normalized to a day. The jump component is of finite activity of the form $J_t^{(i)} = \sum_{s=1}^{N_t^{(i)}} \gamma_{\tau_s^{(i)}}$, for $i = 1, \dots, N$, where $N_t^{(i)}$ is a non-explosive counting process and $\gamma_{\tau_s}^{(i)}$ are jump sizes at times $\tau_s^{(i)}$. Finally, we assume that jump sizes are such that $\forall s = 1, \dots,$ we have $\mathbb{P}\left[\gamma_{\tau_s^{(i)}}^{(i)} = 0\right] = 0$, $i = 1, \dots, N$.

We estimate the realized covariance (e.g. Barndorff-Nielsen and Shephard, 2004a) of the process as follows:

$$RC_t = \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} \left(\Delta_j^n X\right)' \left(\Delta_j^n X\right) \xrightarrow{\mathbb{P}} \int_0^t \Sigma_s ds + \sum_{0\leq s\leq t} \left(\Delta X_s\right)' \left(\Delta X_s\right), \tag{2}$$

where $\Delta_j^n X = \left(\Delta_j^n X^{(i)}\right)_{i=1,\dots,N}$ is an N-dimensional vector containing the jth intraday return, and $\Delta_j^n X^{(i)} = X_{j\Delta_n}^{(i)} - X_{(j-1)\Delta_n}^{(i)}$, where $j = 1, \ldots, n$, $\Delta_n = 1/n$ is the sampling interval, and n is the number of high frequency increments per day. ΔX_s denote the N-dimensional vector of jumps occurring at time s, if a jump occurred, and set to zero if no jump occurred at time s.[4]

In presence of only idiosyncratic jumps the matrix of common jumps has a spherical form with (some) non-zero diagonal elements representing the univariate jumps of each stock. By contrast, when stocks co-jump, the matrix of common jumps is formed by non-zero elements. We estimate the integrated covariation (IC) using the threshold realized covariance estimator of Mancini and Gobbi (2012). This estimator is the multivariate extension of the so-called threshold realized variance (e.g. Jacod, 2008; Mancini, 2001, 2009):

$$TRC_t = \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} \left(\Delta_j^n X \cdot \mathbb{1}_{\{|\Delta_j X| \leq \upsilon_n\}}\right)' \left(\Delta_j^n X \cdot \mathbb{1}_{\{|\Delta_j X| \leq \upsilon_n\}}\right) \xrightarrow{\mathbb{P}} \int_0^t \Sigma_s ds, \tag{3}$$

where $\upsilon_n = \left(\upsilon_n^{(i)}\right)_{i=1,\ldots,N} = \alpha^{(i)} \Delta_n^{\varpi}$, for $\alpha^{(i)} > 0$ and $\varpi \in (0, 1/2)$.[5] The multivariate jump matrix can then be obtained as the difference between the realized covariance and the threshold realized covariance as follows:

$$MJ_t = RC_t - TRC_t \xrightarrow{\mathbb{P}} \sum_{0 \leq s \leq t} (\Delta X_s)' (\Delta X_s). \tag{4}$$

Since we are interested in cases where all the elements of MJ_t are different from zero, i.e. presence of common jumps, we employ the test of Caporin et al. (2017) to identify only the common jumps that are significantly different from zero.

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[4] To illustrate, if in a portfolio, comprised by three assets, all the assets jump jointly, we have that $\Delta X_s = \Delta X_s^{(1)} \Delta X_s^{(2)} \Delta X_s^{(3)} = \gamma_{\tau_s}^{1(23)} \gamma_{\tau_s}^{2(13)} \gamma_{\tau_s}^{3(12)} \Delta_n N_t^{123}$.

[5] As it is customary in the literature (e.g. Aït-Sahalia and Jacod, 2014; Todorov and Bollerslev, 2010; Todorov and Tauchen, 2014, among others), we choose $\alpha^{(i)} = 3\sqrt{BV_t^{(i)}}$ and $\varpi = 0.49$. $BV_t^{(i)} = \mu_1^{-2} \frac{n}{n-1} \sum_{j=2}^{\lfloor 1/\Delta_n \rfloor} |\Delta_j^n X^{(i)}||\Delta_{j-1}^n X^{(i)}|$ is the so-called bipower variation of Barndorff-Nielsen and Shephard (2004b), and $\mu_1 \equiv \mathbb{E}[|\mathcal{N}(0,1)|] = \sqrt{2/\pi}$.

2.1 Multi-jump Test

The multi-jump test of (Caporin et al., 2017, CKR, hereafter) is defined in the following sets:

$$\Omega_t^{MJ,N} = \left\{ \omega \in \Omega \Big| \prod_{i=1}^{N} \Delta X_s^{(i)} \neq 0 \right\}$$
$$\overline{\Omega}_t^N = \Omega \backslash \Omega_t^{MJ,N}.$$

The set $\Omega_t^{MJ,N}$ contains all the trajectories with multi-jumps among all N assets, whereas the complementary set $\overline{\Omega}_t^N$ contains trajectories without multi-jumps in N stocks. However, it can contain jumps and multi-jumps up to $N-1$ stocks. Therefore, the null and alternative hypotheses are:

$$H_0 : (X_t(\omega))_{t\in[0,t]} \in \overline{\Omega}_t^N \quad \text{v.s.} \quad H_1 : (X_t(\omega))_{t\in[0,t]} \in \overline{\Omega}_t^{MJ,N}.$$

The CKR test is based on two jump-robust integrated variance estimators, which generalize the truncated realized variance estimator of Mancini (2001, 2009), and are named smoothed realized variance (SRV). The first SRV takes the following form:

$$SRV_t = \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} |\Delta_j^n X^{(i)}|^2 \cdot K\left(\frac{\Delta_j^n X^{(i)}}{H_{j\Delta_n,n}^{(i)}}\right), \tag{5}$$

where $X^{(i)}$ and $H^{(i)}$ are the respective i-th component of the vectors X and H. $K(\cdot)$ is kernel estimator,[6] and $H_{t,n}$ is the bandwidth defined as:

$$H_{t,n}^{(i)} = h_n \cdot \hat{\sigma}_t^{(i)} \sqrt{\frac{T}{n}}, \tag{6}$$

where $\hat{\sigma}_t^{(i)}$ is a point estimator of the local standard deviation of the i-th stock, $i = 1, \ldots, N$. h_n is the bandwidth parameters, where its role is to gauge the largeness of

[6] As pointed out by the authors, when the kernel function is $K(x) = \mathbb{1}_{|x|\leq\epsilon}$ this estimator is equivalent to the truncated realized variance of Mancini (2001, 2009).

high-frequency returns with respect to the local volatility.[7]

The second SRV is outlined as:

$$\widetilde{SRV}_t^N = \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} |\Delta_j^n X^{(i)}|^2 \cdot \left(K\left(\frac{\Delta_j^n X^{(i)}}{H_{j\Delta_n,n}^{(i)}}\right) + \prod_{k=1}^{N}\left(1 - K\left(\frac{\Delta_j^n X^{(k)}}{H_{j\Delta_n,n}^{(k)}}\right)\right)\right). \tag{7}$$

Returns in the above estimator are smoothed twice. Whereas the first term, $K\left(\frac{\Delta_j^n X^{(i)}}{H_{j\Delta_n,n}^{(i)}}\right)$, has the same effect as in the first SRV, the second term, $\left(1 - K\left(\frac{\Delta_j^n X^{(k)}}{H_{j\Delta_n,n}^{(k)}}\right)\right)$, leaves the corresponding return similar to the raw returns when all multivariate returns are big. Although both smoothing procedures are meant to eliminate jumps, the smoothing in the second SRV allows multi-jumps to survive.

The test statistic is based on the difference between the two SRV estimators. In the absence of multi-jumps, this difference tends to zero, while under the alternative of multi-jumps this difference becomes large and positive. However, the authors need to randomize one of them to obtain a non-degenerate limit distribution under the null. To do so, they apply the wild bootstrap technique suggested in Podolskij and Ziggel (2010), and replace the first SRV by the smoothed randomized realized variance (SRRV):

$$SRRV_t = \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} |\Delta_j^n X^{(i)}|^2 \cdot K\left(\frac{\Delta_j^n X^{(i)}}{H_{j\Delta_n,n}^{(i)}}\right) \cdot \eta_j^i, \tag{8}$$

where $\left(\eta_j^i\right)_{1\leq i\leq N, 1\leq j\leq n}$ is an $N \times n$ matrix of independent and identically distributed (i.i.d.) draws with $\mathbb{E}[\eta_j^i] = 1$ and $\text{Var}[\eta_j^i] = V_n < \infty$. In our application we follow the authors by allowing η_j^i to take values in $\{1+\tau, 1-\tau\}$ with equal probability, so that $V_n = \tau^2$. We set $\tau = 0.05$ so that, in practice, SRRV is virtually indistinguishable from SRV.

[7]We follow the authors and use the same h_n across all the stocks, as the normalization is respect to each stock volatility. As pointed out by Caporin et al. (2017), the advantage of replacing the indicator function with a kernel is that it provides an estimator that depends smoothly on the bandwidth, which stabilizes the procedure in small samples.

The test statistics to detect multi-jumps is described as:

$$S_t^N = \frac{1}{V_n} \sum_{i=1}^{N} \frac{(SRRV_{t,i} - \widetilde{SRV}_{t,i}^{N})^2}{SQX_{t,i}}, \tag{9}$$

where:

$$SQX_t = \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} |\Delta_j^n X^{(i)}|^4 \cdot K^2 \left(\frac{\Delta_j^n X^{(i)}}{H_{j\Delta_n,n}^{(i)}} \right), \quad i = 1, \ldots, N. \tag{10}$$

The authors show that if $\left(\eta_j^i\right)_{1 \le i \le N, 1 \le j \le n}$ is pairwise independent, as $n \to \infty$, it holds that:

$$\begin{cases} S_t^N \xrightarrow{d} \chi_N^n, & on\ \bar{\Omega}_T^N, \\ S_t^N \xrightarrow{d} +\infty & on\ \Omega_T^{MJ,N} \end{cases}, \tag{11}$$

where χ_N^2 denotes the χ-square distribution with N degrees of freedom.

2.2 Disentangling the Continuous and Discontinuous Component

The common jump estimator in equation (4) provides consistent estimates of the multivariate jump component as $\Delta_n \to 0$. In practice, the difference between the realized and threshold realized covariance can be non-zero owing to finite sample problems. As we are interested only in cases where the full multivariate jump matrix is non-zero, we use the multi-jump test (defined above), to disentangle significant common jumps as follows:

$$\begin{aligned} IC_t &= (1 - Z_t) \cdot RC_t + Z_t \cdot TRC_t, \\ CJ_t &= RC_t - C_t, \end{aligned} \tag{12}$$

where $Z_t = \mathbb{1}\left(S_t^N > z_\theta\right)$ and $\mathbb{P}(\chi_N^2 > z_\theta) = \theta$, for $\theta \in (0,1)$. In this paper we use $\theta = 0.01$, and $h_n = h_0 + \frac{c}{N}$, where $h_0 = 1.4$ and $c = 9.57$.[8] The matrix IC_t is equal to the realized covariance when there are no common jumps on day t, while in the presence of common jumps IC_t is equal to the threshold realized covariance (e.g. Andersen et al., 2007a, for a similar approach in a univariate framework). We ignore the impact of idiosyncratic

[8]Those values are recommended by the authors as conservative choices.

jumps,[9] as they have a negligible effect in large portfolios. This is because a specific non-major financial news might be perceived as either good or bad news across some stocks, resulting in respective positive and negative jumps, whose impact is offset in large portfolios.

3 Forecasting Models and Evaluation Criteria

Following Chiriac and Voev (2011),[10] we use the vector heterogeneous autoregressive (vech-HAR) model to estimate and forecast the realized covariance matrix. This model extends the so-called HAR model of Corsi (2009), so that realized covariance is expressed as a linear combination of past daily, weekly and monthly covariances:

$$S_{t+h} = \theta_0 + \theta_d S_t + \theta_w S_{t-5|t} + \theta_m S_{t-22|t} + \epsilon_{t+h}, \tag{13}$$

where $S_t \equiv \text{vech}(RC_t)$ denote the $N^* = N(N+1)/2$ dimensional vectorized version of the realized covariance matrix of interest RC_t. $S_{t-h|t} = \frac{1}{h}\sum_{i=1}^{h} S_{t-i}$ denote the vectorized version of the h-day realized covariance matrix. The intercept θ_0 is an $N^* \times 1$ dimensional vector, while θ_d, θ_w and θ_m parameters are all assumed to be scalar. This simply and extremely parsimonious specification ensures that the covariance matrix forecasts are positive definite.[11]

The standard vech-HAR formulation in equation (13) does not distinguish between Brownian correlation and common jumps. In order to capture those different sources of risk, they must be separately modelled, so that the distinct explanatory factors have their own coefficients. To achieve this, we propose two simple modifications in the spirit

[9] Aït-Sahalia et al. (2020) find that idiosyncratic jumps are related to stock specific events, such as earning disappointments.

[10] Similar to the HAR model (Corsi, 2009), the vech-HAR model approximates long-memory in a parsimonious way. The model involves a fixed number of parameters regardless of the number of assets, which makes it very easy-to-estimate. This model features in the work of Bauer and Vorkink (2011); Bollerslev et al. (2018); Hautsch et al. (2015), inter alios.

[11] A generalization of the vech-HAR model considers that each variance-covariance term has its own dynamics. Therefore, each element in the covariance is estimated separately, which increases the number of parameters from 4 to $4 \times N^*$. As a result, these models yield to forecasts which are not positive definite, and most of the time lead to worse forecasting performance, especially when the number of assets is large.

of Andersen et al. (2007a). The first specification extends the vech-HAR model by incorporating a daily measure of the common jumps, this specification is the vech-HARJ model:

$$S_{t+h} = \theta_0 + \theta_d S_t + \theta_w S_{t-5|t} + \theta_m S_{t-22|t} + \theta_{J_d} J_t + \epsilon_{t+h}. \tag{14}$$

The second specification uses the continuous and multi-jump parts of the realized covariances as shown in Section 2.2. This structure fully incorporates both Brownian correlation and common jumps, because the information content of idiosyncratic jumps remaining in the continuous part is negligible when the number of assets is large. The vech-HARCJ model is outlined as follows:

$$S_{t+h} = \theta_0 + \theta_{C_d} C_t + \theta_{C_w} C_{t-5|t} + \theta_{C_m} C_{t-22|t} + \theta_{J_d} J_t + \theta_{J_w} J_{t-5|t} + \theta_{J_m} J_{t-22|t} + \epsilon_{t+h}, \tag{15}$$

where $J_t \equiv \text{vech}(CJ_t)$ and $C_t \equiv \text{vech}(IC_t)$.

We evaluate the forecasting capability of models based on the Frobenius distance, L_t^F, which extends the mean squared error loss function to the multivariate space, and the Euclidean loss function, L_t^E, computed by equally-weighting all the unique elements of the forecast error matrix:

$$L_t^F = \sqrt{\text{Tr}\left[\left(\widehat{S}_t - RC_t\right)\left(\widehat{S}_t - RC_t\right)'\right]}, \tag{16}$$

$$L_t^E = \sqrt{\text{vech}\left(\widehat{S}_t - RC_t\right)' \text{vech}\left(\widehat{S}_t - RC_t\right)}, \tag{17}$$

where $\widehat{S}_t$ denote the fitted covariance matrices, and RC_t is the ex-post realized covariances. As discussed in Laurent et al. (2013) and Patton (2011b) the ranking produced by both loss functions based on covariance proxies is consistent with those based on the true latent covariance matrix.

We employ the conditional predictive ability (CPA) test of Giacomini and White (2006), to identify models whose losses are significantly smaller than those of the vech-HAR model. Although this approach was developed to asses forecasts in the univariate setting, it directly translates into a multivariate setting when the loss function generates a scalar measure.

4 Monte Carlo Evidence

Using the setup of Barndorff-Nielsen et al. (2011), we simulate a multivariate factor stochastic volatility model for $X_{i,t}$, $i = 1, \ldots, 10$ as:

$$
\begin{aligned}
dX_t^{(i)} &= \mu^{(i)} dt + \sigma_t^{(i)} \left(\rho^{(i)} dB_t^{(i)} + \sqrt{1 - (\rho^{(i)})^2} dW_t \right) + dq_t^{(i)} J_t^{(i)} + dk_t L_t^{(i)} \\
\sigma_t^{(i)} &= \exp\left\{ \beta_0 + \beta_1 \nu_t^{(i)} \right\}, \\
d\nu_t^{(i)} &= \alpha \nu_t^{(i)} dt + dB_t^{(i)}.
\end{aligned}
\tag{18}
$$

The elements of $B_t^{(i)}$ are independent standard Brownian motions and are also independent of W_t. Following Barndorff-Nielsen et al. (2011), we set the parameters to $(\mu, \beta_0, \beta_1, \alpha, \rho) = (0, -5/16, 1/8, -1/40, -0.83)$. The true spot correlation of $X_t^{(i)}$ and $X^{(k)}$ is constant and equals $\sqrt{\left(1 - (\rho^{(i)})^2\right)\left(1 - (\rho^{(k)})^2\right)}$ for $i \neq k$. The leverage between $X_t^{(i)}$ and $\nu_t^{(i)}$ is $\rho^{(i)}$. The fact that ρ is set equal for all i leads to an equicorrelation structure with common correlation of 0.31.[12] The stationary distribution of ν_t is used to restart the process each day at $\nu_0^{(i)} \sim \mathcal{N}\left(0, \left(-2\alpha^{(i)}\right)^{-1}\right)$.

The idiosyncratic jumps are modelled as independent compound Poisson processes, $dq_t^{(i)}$, with intensity $\lambda_J = 0.2$, and jump sizes $\mathcal{N}(0, 0.628)$. The common jumps are determined by a unique compound Poisson process, dk_t, with jump intensity $\lambda_L = 0.1$ and jump sizes $\mathcal{MN}(0, \Pi)$, where $\Pi = \operatorname{diag}(\varrho) \mathit{\Gamma} \operatorname{diag}(\varrho)$. $\mathit{\Gamma}$ is an equicorrelation matrix with common correlation of 0.75, where ϱ is the diagonal matrix containing the standard deviations, which are equal to 0.756. The idiosyncratic and common jump intensities are chosen such that all discontinuities account for about 30% of the sample. As we simulate $T = 2{,}000$ days, there are approximately 600 jumps in total, from which about 200 are common jumps. The jump sizes are set to account for about 30% of the total quadratic

[12] This level of correlation among assets is similar to that found across the asset used in our empirical analysis.

variation of each asset, with common jumps contributing up to 10% (out of the 30%).

The price process is simulated via an Euler scheme, and we normalize one second to be $\Delta_n = 1/23400$, so that the interval $[0, 1]$ contains 6.5 hrs. In generating the observed price, we discretize $[0, 1]$ into a number $n = 23,400$ of intervals. The prices are then aggregated to the 5-min, which is equivalent to 78 observations per day.[13] The forecasts are based on a rolling window of 500 days and 1,000 replications.[14]

Table 1 reports the simulated daily, weekly and monthly in- and out-of-sample forecasts of the HAR, HARJ, and HARCJ models. The results of Panels A and B are based on common jumps; and those in Panels C and D are based on directional common jumps (see Section 6 for details about the estimation of directional common jumps). As shown in Panels A and B, the incorporation of common jumps not only improves the in-sample fits of the realized covariances, but also the out-of-sample forecast accuracy. The use of directional common jumps also improves on the in- and out-of-sample forecasts of the HAR model irrespective of the forecast horizon.

Across all forecast horizons, the HARCJ model consistently outperforms both the HAR and HARJ models, with forecasting gains increasing slightly as the horizon lengthens. This result suggests that separating out these two sources of risk increases the persistence of the models relative to that of the HAR model. Hence, the inference that models explicitly accounting for the presence of common jumps provide more accurate predictions of covariance matrices at longer horizons.

5 Empirical Results

5.1 Data

We consider 20 individual Dow Jones stocks from the period 2000–2016. The data are sourced from the TickData database. We use the previous tick interpolation to aggregate the data down to the required sampling frequency. We sample returns every 5-minutes,

[13] Monte Carlo results for 10- and 15-min are qualitatively similar to those obtained at the 5-min sampling, and therefore not reported here.

[14] The rolling window size in our simulation represents 25% of the full sample size ($T = 2,000$), which is equivalent to the rolling window size used in our empirical study.

which results in 78 observations per day. This sampling frequency is customary in the high-frequency literature, as it provides a trade-off between achieving enough statistical power and avoiding distortions that may arise from microstructure noise (e.g. Hansen and Lunde, 2006; Patton, 2011a). To validate the latter statement, we perform the Hausman test for the presence of market microstructure noise of Aït-Sahalia and Xiu (2019). We use the third test ($H_{3,n}$) proposed by the authors as this test is robust to jumps, and as a robustness check we employ the first-order autocorrelation in log-returns test (AC_n).[15] The last two columns of Table 2 report the proportion of rejections of both $H_{3,n}$ and AC_n across all individual stocks. The average proportion of rejections is respectively 0.025 and 0.017 for $H_{3,n}$ and AC_n, suggesting that the level of microstructure noise in our dataset is negligible at the 5-minutes sampling frequency, and therefore we can treat our dataset as noise-free.

Table 2 reports the descriptive statistics of the 20 individual stocks together with the number of identified common jumps and their contribution to the total variance. The second panel shows the descriptive statistics of the common jumps – based on days where the null of no simultaneous jumps is rejected. American Express (AXP) and JP Morgan Chase (JPM) display the highest average (co)jump size, with peaks detected during the global financial crisis. The contribution of common jumps to the total variance, displayed in the last column of Table 2, shows values ranging between 1.4–5%, with an average of 2.2%. Previous findings in the literature document a contribution of jumps to the total variance of about 6% (Huang and Tauchen, 2005), which means that common jumps make up a significant portion of the total jump part.

Table 3 displays in the lower (upper) triangular section, the correlations of the common jumps (continuous component) across all the individual stocks. Unlike idiosyncratic jumps, the simultaneous arrival of common jumps triggered by the wide-market economic information results in highly correlated jump measures, with an average correlation of 0.67. Surprisingly, the level of correlation found in common jumps is slightly higher than that of the continuous component whose average correlation is 0.65. This finding is in

[15] These two tests are recommended by the authors, however our results are qualitatively similar under the alternative Hausman tests proposed by Aït-Sahalia and Xiu (2019).

line with Das and Uppal (2004) and Longin and Solnik (2001) who note that the increase in correlation, after a collective crash, is mainly explained by the occurrence of common jumps.

5.2 Common Jumps and the Link with Major Financial and Economic News

Over 149 days, our empirical results show stocks to have jumped simultaneously.[16] Our results indicate that over 85% of these common jumps are strongly associated with major financial and economic news. However, days with common jumps account for less than 5% of the total major financial and economic news available for the period under analysis. For instance, FOMC meetings are scheduled every 6 weeks, which means, between 2000–2016, at least 136 meetings took place.[17]

To illustrate some of these findings, Figure 1 lists 6 days when the CKR test detected simultaneous jumps: i) May 06, 2010 – in a flash crash (aka the crash of 14:45 hrs) US stocks lost one trillion dollar in 36 minutes, with the Dow Jones losing 998.5 points or 9%. However, the loses were rapidly recovered; ii) April 23, 2013 – a flash crash was associated with a false report of White House explosions. This news triggered a 143-point fall in the Dow Jones. However, the fake tweet was immediately corrected, allowing the stock market to recover the big losses within minutes; iii) February 03, 2016 – US stock market indices gains reflected an 8% rise in oil prices as Russia and OPEC cut production. This effort translated in 183 points gained by S&P 500 index; iv) November 06, 2002 – The Fed cut both the target federal reserve funds rate, and overnight bank lending rate, by half a percentage point to 1.25 percent, a new 40-year low; v) August 09, 2011 – large stock market gains followed the Fed announcement that interest rates would remain low until 2013; vi) January 27, 2016 – as disappointing quarterly reports renewed concerns

[16]Using different bandwidth settings, Caporin et al. (2017) identify between 20–101 simultaneous jumps.

[17]Other important news are, among others, the Consumer Price index (CPI), Producer Price Index (PPI), Manufacturing Composite Index (MCI) and employee on non-farm payrolls, all released monthly, while DGP-related news are released quarterly. For a full list of the scheduled macroeconomic announcements, the reader can consult Lahaye et al. (2011).

over economic growth the Dow Jones Industrial average closed lower by 222 points.

In line with Lahaye et al. (2011) and Dungey and Hvozdyk (2012), we find that macroeconomic announcements are generally sufficient to produce simultaneous jumps, with returns jumping between 0.8–2.0%. The resulting sign of the simultaneous jumps triggered by these type of news is often negative. This result is consistent with that of Amengual and Xiu (2018), as they find that downward intraday volatility jumps in the S&P 500 index are often associated with a resolution of policy uncertainty, mostly through statements from the FOMC meetings and speeches by the chair of the Federal Reserve. Our results also indicate that simultaneous jumps are realized within 30 minutes around the schedule time.[18] On the other hand, flash crashes or market sell-offs usually spark bigger simultaneous jumps, with sizes between 1.5–5%.[19]

5.3 In-Sample Estimates

Table 4 reports the in-sample parameter estimates with robust standard errors in parentheses. The last three rows report the goodness-of-fit measures for the different models across three forecast horizons: one-day ($h = 1$), one-week ($h = 5$) and one-month ($h = 22$). As expected, the daily, weekly and monthly parameter estimates of the realized covariance and its continuous component are strongly significant across all forecasting horizons. The magnitudes of the parameters are similar across all the models; however, the HAR-J and HAR-CJ give greater weight to the daily and weekly estimates, and thus those forecasting models react faster to new information.

In respect of jump variables, for the HAR-J model the jump estimate is generally negative and significant across all forecasting horizons. This result is in line with the findings obtained in the univariate framework (e.g. Andersen et al., 2007a; Corsi et al., 2010). Contrasting with the results for the univariate literature, where most of the jump estimates of the HARCJ model are negative and insignificant (e.g. Andersen et al.,

[18]The manufacturing report released on July 1, 2011 is an example of another macroeconomic announcement that triggers simultaneous jumps at the released time (10:00 hrs), with returns jumping more than 1%.

[19]Other relevant stock market sell-off identified in our sample are on August 18 and 24 and 25, 2015 due to fear of a lack of liquidity in the market, and on Jun 24, 2016 due to the Brexit Referendum.

2007a), common jumps estimates in the multivariate HARCJ model are generally positive and strongly significant across all forecasting horizons. This supports the view that the information that is implicit from common jumps (increasing both correlation among assets and the persistence of stock (co)variances) can improve the accuracy of multivariate forecasts. The greater persistence from the HARJ and HARCJ directly translates into 'better fits' for in-sample models.

5.4 Out-of-Sample Forecasts

Out-of-sample results across three different forecast horizons are shown in Table 5. Bold numbers indicate losses of the HARJ and HARCJ models outperforming those of the HAR models. Starred p-values of the CPA test of Giacomini and White (2006) indicate that losses of the HARJ and HARCJ models are smaller than those from HAR model at the 5% significance level.

As with the in-sample results, and as confirmed by the CPA test, the HARJ and HARCJ models consistently outperform those of the HAR model irrespective of the loss function and forecast horizon considered.[20] For instance, the null of equal predictive ability is rejected across all forecasting horizons for the HARCJ and at the one-month horizon for the HARJ model.

The bigger forecasting gains from the HARCJ model are in line with the univariate forecasting literature (e.g. Andersen et al., 2007a; Duong and Swanson, 2015). The associated rationale is as follows: As the HARCJ model relies upon full decomposition of the covariance matrix (continuous and common jumps), it captures the two distinct sources of risk and their different dependencies across various horizons. Moreover, some financial and economic news might have a longer impact in the stocks, and therefore the inclusion of the weekly and monthly common jumps variables increase the predictability of covariance matrices, as the coefficients of these measures capture this residual information.

[20]This finding is also corroborated by our Monte Carlo experiment in Section 4.

6 Directional Common Jumps

This section investigates the incidence of directional common jumps in forecasting realized covariance matrices. Bollerslev et al. (2020) show that the realized covariance matrix may be decomposed into four distinct elements: two based on concordant signs and two based on discordant signs as follows:[21]

$$\begin{aligned} \mathtt{P}_t &= \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} p\left(\Delta_j^n X\right)' p\left(\Delta_j^n X\right), \quad \mathtt{N}_t = \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} n\left(\Delta_j^n X\right)' n\left(\Delta_j^n X\right), \\ \mathtt{M}_t &= \sum_{j=1}^{\lfloor 1/\Delta_n \rfloor} \left(p\left(\Delta_j^n X\right)' n\left(\Delta_j^n X\right) + n\left(\Delta_j^n X\right)' p\left(\Delta_j^n X\right)\right), \end{aligned} \tag{19}$$

where $p(x) \equiv \max\{x, 0\}$ and $n(x) \equiv \min\{x, 0\}$ denote the component-wise positive and negative elements of the real vector x. Therefore, $\mathtt{P}_t$ and $\mathtt{N}_t$ correspond to the positive and negative realized semicovariance matrices, while $\mathtt{M}_t$ is the sum of the two discordant elements. In the presence of jumps, these three elements contain both diffusive and jump covariation components. As shown in Bollerslev et al. (2020, Section 2), the limiting behavior of $\mathtt{P}_t$ and $\mathtt{N}_t$, derived for a pair of assets, only differs in the jump component:

$$\mathtt{P}_t \xrightarrow{\mathbb{P}} \int_0^t \phi_{j,s}\phi_{k,s}\psi(\rho_{jk,s})ds + \sum_{0\le s\le t} p\left(\Delta X_s\right)' p\left(\Delta X_s\right), \tag{20}$$

$$\mathtt{N}_t \xrightarrow{\mathbb{P}} \int_0^t \phi_{j,s}\phi_{k,s}\psi(\rho_{jk,s})ds + \sum_{0\le s\le t} n\left(\Delta X_s\right)' n\left(\Delta X_s\right), \tag{21}$$

$$\mathtt{M}_t \xrightarrow{\mathbb{P}} -2\int_0^t \phi_{j,s}\phi_{k,s}\psi(-\rho_{jk,s})ds + \sum_{0\le s\le t} \left(p\left(\Delta X_s\right)' n\left(\Delta X_s\right) + n\left(\Delta X_s\right)' p\left(\Delta X_s\right)\right), \tag{22}$$

where $\phi_{i,t} = \Sigma_{ii,t}^{1/2}$ is the spot volatility of asset i, $\rho_{ik,t} = \Sigma_{ik,t}/(\phi_{j,t}\phi_{k,t})$ denotes the spot correlation coefficient between assets i and k, and $\psi(\rho) = (2\pi)^{-1}\left(\rho \arccos(-\rho) + \sqrt{1-\rho^2}\right)$.[22] Given that the diffusive component of the positive and negative semicovariances is the same, the intuition that these measures carry distinct economic information about the good and bad news over each day resides only in their jump component. Therefore, the

[21] The realized semicovariances proposed by Bollerslev et al. (2020) can be seen as a multivariate extension of the realized semivariance pioneered by Barndorff-Nielsen et al. (2010).

[22] $\psi(\rho)$ corresponds to $\mathbb{E}[Z_1, Z_2 \mathbb{1}_{\{Z_1<0, Z_2<0\}}]$ bivariate standard normally distributed with correlation ρ.

directional common jumps are estimated as follows:[23]

$$DCJ_t = \mathtt{P}_t - \mathtt{N}_t \xrightarrow{\mathbb{P}} p\left(\Delta X_s\right)' p\left(\Delta X_s\right) - n\left(\Delta X_s\right)' n\left(\Delta X_s\right). \tag{23}$$

To illustrate the impact of common jumps in the positive and negative semicovariances, Figure 2 depicts the intraday returns on the day of the common jumps, and the positive and negative semivariances for a five-day trading period around the date of the common jumps. Two subplots (top panel) show the intraday returns (left) on June 29, 2006 together with the positive and negative semicovariances (right) between 26 and 30 June 2006. On June 29, the Fed raised the short-term rate by a quarter-percentage point. There followed a positive jump in the S&P 500 of about 0.7% intraday and a daily change of 2.2%. This positive simultaneous jumps is fully absorbed by the positive semicovariance, and therefore its magnitude is much bigger than that of the negative semicovariance.

The bottom panel plots the intraday returns (left) on August 5, 2014, and the concordant elements of the realized covariance (right) during the week beginning on August 4, 2014. On August 5, 2014, Russian troops were reported lining on the borders of Ukraine. This news triggered a negative simultaneous jumps, which is completely absorbed by the negative semicovariance as shown in Figure 2. This evidence suggests that the information content and dynamic dependencies of the directional common jumps might improve the forecasting accuracy of realized covariance matrices.

6.1 In-Sample Estimates

Table 6 reports in-sample parameter estimates (robust standard errors in parentheses) for daily ($h = 1$), weekly ($h = 5$) and monthly ($h = 22$) forecasts, together with the respective measures of fit. As with the results in Table 4, the parameter estimates of the realized covariance and its continuous component are strongly significant across all forecasting horizons. However, directional common jumps show a different pattern to that

[23]Significant directional common jumps are identified by intersecting this measure with the indicator function of the CKR test as, $DCJ_t \cdot Z_t$.

of common jumps. The estimates are generally significantly negative across all horizons. As negative/positive common jumps are fully absorbed by the negative/positive semicovariances, their difference reflects the direction of the common jumps, and therefore the negative estimates indicate that common jumps increase the persistence and future level of the (co)variance. This finding corroborates those of Patton and Sheppard (2015), that negative semivariances are more important than their positive counterparts for modelling and forecasting in the univariate framework.

The measures of fit (Table 6, last 3 rows) indicate that the use of directional common jumps increases the in-sample fitting of realized covariance matrices. Larger improvements are at longer horizons, with the HARCJ model achieving the best in-sample performance.

6.2 Out-of-Sample Forecasts

Out-of-sample forecast results using directional common jumps are reported in Table 7. The losses of the HARJ and HARCJ models (indicated by bold numbers) are smaller than those of the HAR models. For the CPA test p-values (Giacomini and White, 2006), starred values indicate (at the 5% significance level) significantly smaller losses than those of the HAR model. This is wholly consistent with our analysis. The biggest out-of-sample gains are attined by the HARCJ models, which outperform both the HAR and HARJ models irrespective of the forecast horizons and loss function under analysis. However, the CPA test indicates that only the losses of the HARCJ model, at the one-day ($h = 1$) and one month ($h = 22$) horizons, are significantly smaller than those of the HAR models. Whereas at the one-week ($h = 5$) horizon the losses of the HAR model are significantly smaller (10% level) to those of the HAR model, the HARJ model fails to outperform the HAR model across all horizons. This suggests that, although directional common jumps improve the predictability of the realized covariance matrices, their contribution is more limited than that of common jumps.

7 Minimum Variance Portfolios

This section assesses the economic value of the different models by constructing Global Minimum Variance (GMV) portfolios. The GMV approach relies solely upon returns covariances, which makes it a 'clean' framework for evaluating the merits of the different covariance forecasts. This is because the estimation errors in sample means are large and the corresponding portfolios perform poorly compared to the GMV portfolio (e.g. DeMiguel et al., 2009; Jagannathan and Ma, 2003). We employ daily, weekly and monthly rebalancing frequencies, and in each period the investor solves the following minimization problem:

$$\begin{aligned} w_t^* = \arg\min_{w_t} \quad & w_t' \widehat{S}_t w_t, \\ \text{s.t} \quad & w_t' \iota = 1, \end{aligned} \tag{24}$$

where w_t is an $N \times 1$ vector of GMV portfolio weights, ι is an $N \times 1$ vector of ones, and $\widehat{S}_t$ is the $N \times N$ matrix of forecasted covariances from a particular mode. The optimal portfolio weights, w_t^*, are given by:

$$w_t^* = \frac{\widehat{S}_t^{-1} \iota}{\iota' \widehat{S}_t^{-1} \iota}. \tag{25}$$

It is well-known that inaccurate estimates of the covariance matrix lead to worse portfolio performance, with higher turnover and trading costs (e.g. DeMiguel et al., 2014; Han, 2006). Thereby, we incorporate these features in our analysis and define the total portfolio turnover from day t to day $t+1$ as:

$$TO_t = \sum_{i=1}^{N} \left| w_{t+1}^{*(i)} - w_t^{*(i)} \frac{1 + r_t^{(i)}}{1 + w_t^{*'} r_t} \right|. \tag{26}$$

The portfolio excess return net of transaction cost is therefore:

$$r_{p_t} = w_t^{*'} r_t - cTO_t, \tag{27}$$

where c is the transaction cost.

We follow Fleming et al. (2001, 2003), and evaluate the economic significance of the different strategies using a utility-based framework, in which the investor has quadratic utility with risk aversion γ. The realized daily utility generated by the portfolio based on the covariance forecasts from model k is:

$$U\left(r_{p_t}^{(k)},\gamma\right)=\left(1+r_{p_t}^{(k)}\right)-\frac{\gamma}{2(\gamma+1)}\left(1+r_{p_t}^{(k)}\right)^2. \tag{28}$$

The economic value of the different models can be determined by solving:

$$\sum_{t=1}^{T}U\left(r_{p_t}^{(k)},\gamma\right)=\sum_{t=1}^{T}U\left(r_{p_t}^{(q)}-\Delta_\gamma,\gamma\right), \tag{29}$$

where Δ_γ can be interpreted as the return an investor with risk-aversion γ is willing to sacrifice to switch from using model k to using model q.

The GVM strategies, using a risk-aversion $\gamma=6$ and a transaction cost $c=0.5\%$, for the different models based on daily, weekly and monthly rebalancing are reported in Table 8. We use a dagger (†) to differentiate the HARJ and HARCJ models that utilize the directional common jumps, and the starred values indicate that Δ_6 is significantly different from zero.[24]

The results in Table 8 show that separating common jumps from the continuous component not only increases the accuracy of the forecasted covariances, but leads to substantial gains in portfolio performance. This is because the increased accuracy of the forecasted covariances leads to more stable portfolio strategies, thereby reducing the trading costs. Although the HARJ and HARCJ strategies, using both common and directional common jumps, improve on the performance of the HAR model, the portfolio gains of the HARCJ strategy are significantly different from zero across all horizons. In line with our previous results, common jumps provide superior performance than directional common jumps.

[24]To evaluate the performance of our strategies, we follow Bandi et al. (2008) and Engle (2002) and create a null hypothesis that examines whether the performance is equal to zero, i.e. $H_0:\Delta_6=0$ and $H_1:\Delta_6>0$. Therefore, we apply a one-sided t-test with a robust variance-covariance estimator.

8 Conclusion

We propose a robust non-parametric framework that builds on the recent theoretical developments of Barndorff-Nielsen and Shephard (2004a), Mancini and Gobbi (2012), and Caporin et al. (2017), and provide an easy-to-implement approach for measuring separately the multivariate continuous and common jumps components of quadratic covariation matrices. We further investigate the incidence of directional common jumps, which are estimated as the difference between the positive and negative semicovariances. The sign of the directional common jumps depends upon the direction of the simultaneous jumps, as the latter are fully absorbed by the positive or negative semicovariances (e.g. Bollerslev et al., 2020).

Applying the theory to 20 individual stocks for a period of 17 years, we find that common jumps are strongly associated with major financial and economic news. As common jumps are highly correlated, their occurrence generally increases both the dependence across stocks and the persistence of stock (co)variances.

The inclusion of the continuous and common jumps in a vectorized heterogeneous autoregressive (vech-HAR) model results in significant in- and out-of-sample forecasting gains, which are attained at the daily, weekly and monthly horizons. When the continuous and the common jumps variables are entered separately in the vech-HAR model, common jumps estimates are generally positive, and lead to an increase in future covariances. On the other hand, estimates of directional common jumps are usually negative, which means that negative (positive) common jumps increase (decrease) the persistence and future level of (co)variances.

Finally, by using a utility-based approach to assess the economic value of the forecasted covariance matrices, we find that the improved accuracy of the HARJ and HARCJ models yields cheaper portfolio allocations, as the more stable portfolio strategies lead to lower trading costs.

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A Tables and Figures

Table 1: Simulated In- and Out-of-sample Forecast Results

	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ
	$h=1$			$h=5$			$h=22$		
	Panel A: In-Sample Forecasts (Common Jumps)								
R^2_{adj}	0.336	0.350	0.365	0.622	0.646	0.674	0.695	0.718	0.746
L^F	2.813	2.770	2.677	1.968	1.921	1.810	1.649	1.599	1.498
L^E	2.376	2.302	2.274	1.666	1.621	1.543	1.442	1.397	1.330
	Panel B: Out-of-Sample Forecasts (Common Jumps)								
L^F	2.827	2.767	2.758	1.977	1.908	1.892	1.657	1.595	1.554
L^E	2.387	2.340	2.316	1.672	1.616	1.577	1.448	1.401	1.376
	Panel C: In-Sample Forecasts (Directional Common Jumps)								
R^2_{adj}	0.336	0.337	0.365	0.622	0.623	0.674	0.695	0.695	0.746
L^F	2.813	2.793	2.704	1.968	1.965	1.829	1.649	1.618	1.513
L^E	2.376	2.346	2.297	1.666	1.650	1.559	1.442	1.412	1.344
	Panel D: Out-of-Sample Forecasts (Directional Common Jumps)								
L^F	2.827	2.802	2.786	1.977	1.979	1.910	1.657	1.659	1.583
L^E	2.387	2.365	2.339	1.672	1.673	1.591	1.448	1.449	1.385

Note: The table reports the in- and out-of-sample results for the different models based on daily ($h = 1$), weekly ($h = 5$) and monthly ($h = 22$) horizons. L^F and L^E denote the Frobenius and Euclidean loss function, respectively. The results are generated using the Monte Carlo simulation outlined in Section 4.

Table 2: Descriptive Statistics

Name	Ticker	Variance		Common Jumps			Common Jumps	Hausman Test	
		Mean	St. Dev.	Mean	St. Dev	Identified	to Total Variance	$H_{3,n}$	AC_n
American Express	AXP	4.020	9.195	2.619	5.495	149	2.178	0.022	0.015
Boeing	BA	2.812	3.900	1.592	2.961	149	1.879	0.029	0.024
Catterpillar	CAT	3.239	4.889	2.081	3.512	149	2.133	0.022	0.016
Disney	DIS	2.966	5.015	1.973	4.462	149	2.177	0.030	0.023
Dow	DOW	3.976	7.353	2.601	6.061	149	2.080	0.023	0.019
Dupont	DD	2.761	4.076	1.554	2.814	149	1.843	0.024	0.016
Home Depot	HD	3.121	4.938	2.101	5.592	149	2.188	0.022	0.015
IBM	IBM	2.026	3.527	1.454	4.845	149	2.434	0.019	0.014
Intel	INTC	4.075	5.754	1.867	4.564	149	1.511	0.018	0.015
Johnson & Johnson	JNJ	1.385	3.482	2.047	14.222	149	5.010	0.031	0.019
JP Morgan Chase	JPM	4.615	10.848	4.231	11.760	149	2.937	0.024	0.014
Coca-Cola	KO	1.561	2.535	0.781	1.389	149	1.685	0.027	0.017
MacDonald	MCD	2.164	4.333	0.943	1.850	149	1.417	0.036	0.024
Merck and Co.	MRK	2.431	5.227	1.570	4.004	149	2.219	0.028	0.015
3M	MMM	1.853	3.216	1.297	3.647	149	2.307	0.026	0.017
Microsoft	MSFT	2.679	3.854	1.279	2.014	149	1.630	0.021	0.015
Procter & Gamble	PG	1.492	2.907	1.106	3.332	149	2.445	0.027	0.017
United Technologies	UTX	2.300	3.793	1.497	3.009	149	2.191	0.025	0.017
WalMart	WMT	2.045	3.277	0.992	2.001	149	1.609	0.026	0.018
Exxon Mobil	XOM	1.987	3.955	1.424	3.720	149	2.379	0.020	0.014

Note: The table reports the descriptive statistics of the variance and common jumps together with the contribution of the common jumps to total variance. The latter is estimated as the ratio of sum of the common jumps (of stock i) to the sum of the total variance (of stock i). The descriptive statistics of the common jumps only considers the days with significant common jumps, depicted in the third column. The last two columns report respectively the proportion of rejections of the Hausman test for the presence of market microstructure noise and first-order autocorrelation in log-returns test proposed by Aït-Sahalia and Xiu (2019).

Table 3: The Correlation of the Common Jumps and Continuous Components

	AXP	BA	CAT	DIS	HD	IBM	INTC	JNJ	JPM	KO	MCD	MRK	MMM	MSFT	PG	UTX	WMT	XOM	DD	DOW
AXP	–	0.699	0.822	0.693	0.792	0.752	0.633	0.662	0.802	0.686	0.555	0.539	0.796	0.681	0.610	0.715	0.653	0.802	0.791	0.554
BA	0.554	–	0.754	0.745	0.795	0.742	0.697	0.743	0.639	0.727	0.540	0.491	0.776	0.737	0.616	0.798	0.689	0.727	0.761	0.492
CAT	0.722	0.854	–	0.686	0.773	0.774	0.632	0.692	0.717	0.758	0.528	0.561	0.820	0.721	0.671	0.777	0.682	0.808	0.846	0.593
DIS	0.603	0.820	0.845	–	0.755	0.715	0.696	0.704	0.629	0.679	0.556	0.467	0.740	0.705	0.586	0.751	0.666	0.702	0.695	0.489
HD	0.678	0.517	0.611	0.541	–	0.809	0.740	0.757	0.716	0.774	0.584	0.537	0.820	0.769	0.662	0.796	0.776	0.785	0.800	0.505
IBM	0.833	0.531	0.630	0.538	0.570	–	0.821	0.728	0.688	0.766	0.566	0.506	0.798	0.829	0.697	0.755	0.806	0.755	0.779	0.504
INTC	0.496	0.877	0.773	0.810	0.431	0.573	–	0.679	0.604	0.705	0.532	0.433	0.692	0.848	0.620	0.677	0.776	0.600	0.679	0.474
JNJ	0.299	0.253	0.280	0.280	0.858	0.193	0.192	–	0.635	0.769	0.583	0.519	0.751	0.724	0.665	0.746	0.728	0.753	0.734	0.485
JPM	0.802	0.635	0.722	0.556	0.658	0.765	0.427	0.276	–	0.629	0.501	0.500	0.674	0.652	0.530	0.622	0.594	0.680	0.692	0.553
KO	0.760	0.667	0.842	0.682	0.739	0.580	0.469	0.443	0.821	–	0.583	0.559	0.775	0.737	0.717	0.772	0.775	0.756	0.770	0.476
MCD	0.700	0.558	0.756	0.676	0.847	0.544	0.486	0.669	0.626	0.819	–	0.384	0.598	0.542	0.519	0.572	0.590	0.602	0.560	0.331
MRK	0.498	0.850	0.831	0.827	0.668	0.430	0.824	0.521	0.506	0.665	0.711	–	0.541	0.482	0.458	0.498	0.480	0.560	0.555	0.396
MMM	0.426	0.866	0.817	0.814	0.411	0.411	0.865	0.233	0.420	0.537	0.526	0.898	–	0.762	0.717	0.837	0.749	0.854	0.833	0.528
MSFT	0.677	0.676	0.667	0.592	0.819	0.623	0.631	0.662	0.606	0.689	0.718	0.674	0.544	–	0.663	0.733	0.762	0.699	0.753	0.496
PG	0.631	0.667	0.705	0.636	0.870	0.570	0.594	0.767	0.612	0.708	0.781	0.822	0.701	0.820	–	0.668	0.706	0.672	0.688	0.425
UTX	0.639	0.833	0.863	0.779	0.624	0.589	0.691	0.276	0.800	0.810	0.686	0.771	0.686	0.605	0.617	–	0.741	0.773	0.787	0.504
WMT	0.761	0.567	0.775	0.607	0.737	0.712	0.542	0.462	0.677	0.782	0.857	0.630	0.556	0.704	0.739	0.671	–	0.697	0.748	0.443
XOM	0.527	0.860	0.861	0.857	0.499	0.476	0.838	0.310	0.496	0.663	0.653	0.917	0.939	0.596	0.739	0.709	0.615	–	0.802	0.481
DD	0.613	0.910	0.883	0.805	0.523	0.524	0.780	0.219	0.707	0.747	0.607	0.804	0.792	0.644	0.622	0.876	0.554	0.818	–	0.570
DOW	0.542	0.790	0.743	0.713	0.722	0.479	0.779	0.607	0.489	0.583	0.645	0.853	0.757	0.810	0.812	0.633	0.552	0.760	0.750	–

Note: The table reports in the lower (upper) triangular matrix the correlation of the common jumps (continuous component) across the 20 individual stocks using the full sample size.

Table 4: In-Sample Estimates using Common Jumps

	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ
		$h = 1$			$h = 5$			$h = 22$	
θ_0	0.087	0.084	0.092	0.133	0.130	0.139	0.241	0.240	0.242
s.e.	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.008)	(0.008)	(0.008)
θ_d/θ_{C_d}	0.255	0.294	0.268	0.170	0.207	0.195	0.097	0.102	0.111
s.e.	(0.018)	(0.023)	(0.023)	(0.006)	(0.007)	(0.007)	(0.004)	(0.005)	(0.005)
θ_w/θ_{C_w}	0.377	0.357	0.431	0.326	0.308	0.341	0.267	0.264	0.238
s.e.	(0.023)	(0.024)	(0.027)	(0.016)	(0.016)	(0.020)	(0.018)	(0.018)	(0.020)
θ_m/θ_{C_m}	0.281	0.275	0.204	0.371	0.365	0.310	0.396	0.395	0.402
s.e.	(0.014)	(0.014)	(0.015)	(0.015)	(0.015)	(0.016)	(0.018)	(0.018)	(0.020)
θ_{J_d}		−0.285	0.119		−0.267	−0.018		−0.033	0.022
s.e.		(0.030)	(0.016)		(0.015)	(0.016)		(0.021)	(0.007)
θ_{J_w}			−0.487			−0.170			0.490
s.e.			(0.055)			(0.062)			(0.077)
θ_{J_m}			1.443			1.450			0.436
s.e.			(0.148)			(0.182)			(0.113)
R^2_{adj}	0.517	0.520	0.525	0.643	0.647	0.651	0.609	0.609	0.610
L^F	14.998	14.978	14.861	11.509	11.491	11.429	15.779	15.742	15.652
L^E	11.867	11.852	11.761	9.188	9.174	9.127	12.469	12.441	12.371

Note: The table reports the in-sample parameter estimates with robust standard errors in parentheses, along with measures of fit for the different models estimated using forecast horizons equal to one-day ($h = 1$), one-week ($h = 5$) and one-month ($h = 22$). L^F and L^E denote the respective Frobenius and Euclidean loss function. The standard errors are computed using methods that are robust to model misspecification.

Table 5: Out-of-sample Forecast Results using Common Jumps

	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ
		$h=1$			$h=5$			$h=22$	
L^F	13.481	**13.472**	**12.782**	10.720	**10.703**	**10.427**	11.705	**11.443**	**11.354**
CPA	–	0.237	0.001⋆	–	0.746	0.021⋆	–	0.041⋆	0.035⋆
L^E	10.548	**10.543**	**10.011**	8.428	**8.416**	**8.205**	9.044	**8.841**	**8.783**
CPA	–	0.252	0.001⋆	–	0.773	0.014⋆	–	0.042⋆	0.030⋆

Note: The table reports the out-of-sample forecast loss for the different models, along with the p-value of the CPA test of Giacomini and White (2006) based on a 5% significance level. Bold numbers indicate that the forecast losses are smaller than that of the HAR model, while starred numbers highlight the forecasts whose losses are significant smaller relative to the HAR model.

Table 6: In-Sample Estimates using Directional Common Jumps

	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ
		$h=1$			$h=5$			$h=22$	
θ_0	0.087	0.087	0.068	0.133	0.132	0.106	0.241	0.238	0.216
s.e.	(0.007)	(0.007)	(0.008)	(0.008)	(0.008)	(0.009)	(0.007)	(0.007)	(0.008)
θ_d/θ_{C_d}	0.255	0.255	0.272	0.170	0.171	0.189	0.097	0.099	0.101
s.e.	(0.018)	(0.019)	(0.023)	(0.008)	(0.008)	(0.010)	(0.007)	(0.007)	(0.008)
θ_w/θ_{C_w}	0.377	0.377	0.423	0.326	0.327	0.370	0.267	0.268	0.285
s.e.	(0.023)	(0.023)	(0.028)	(0.017)	(0.017)	(0.023)	(0.018)	(0.018)	(0.022)
θ_m/θ_{C_m}	0.281	0.281	0.278	0.371	0.371	0.377	0.396	0.395	0.434
s.e.	(0.014)	(0.014)	(0.016)	(0.016)	(0.016)	(0.019)	(0.016)	(0.016)	(0.019)
θ_{J_d}		−0.026	0.144		−0.071	0.242		−0.201	0.058
s.e.		(0.026)	(0.019)		(0.022)	(0.022)		(0.023)	(0.019)
θ_{J_w}			−0.257			−1.121			−0.950
s.e.			(0.071)			(0.103)			(0.107)
θ_{J_m}			−1.477			−1.437			−1.198
s.e.			(0.182)			(0.203)			(0.174)
R^2_{adj}	0.517	0.517	0.523	0.643	0.643	0.655	0.609	0.610	0.612
L^F	14.998	14.998	14.707	11.509	11.506	11.132	11.584	11.556	11.030
L^E	11.867	11.867	11.662	9.188	9.186	8.916	9.190	9.168	8.774

Note: The table reports the in-sample parameter estimates with robust standard errors in parentheses, along with measures of fit for the different models estimated using forecast horizons equal to one-day ($h = 1$), one-week ($h = 5$) and one-month ($h = 22$). L^F and L^E denote the respective Frobenius and Euclidean loss function. The standard errors are computed using methods that are robust to model misspecification. The jump variable used to generate these models is the directional common jumps estimated as in equation (23). To ease the notation we prefer not to change the name given to the specifications.

Table 7: Out-of-sample Forecast Results using Directional Common Jumps

	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ	HAR	HARJ	HARCJ
		$h = 1$			$h = 5$			$h = 22$	
L^F	13.481	**13.464**	**12.904**	10.720	**10.716**	**10.552**	11.705	**11.649**	**10.355**
CPA	–	0.446	0.011*	–	0.930	0.056	–	0.162	0.037*
L^E	10.548	**10.533**	**10.129**	8.428	**8.424**	**8.328**	9.044	**9.001**	**8.013**
CPA	–	0.422	0.015*	–	0.975	0.097	–	0.156	0.034*

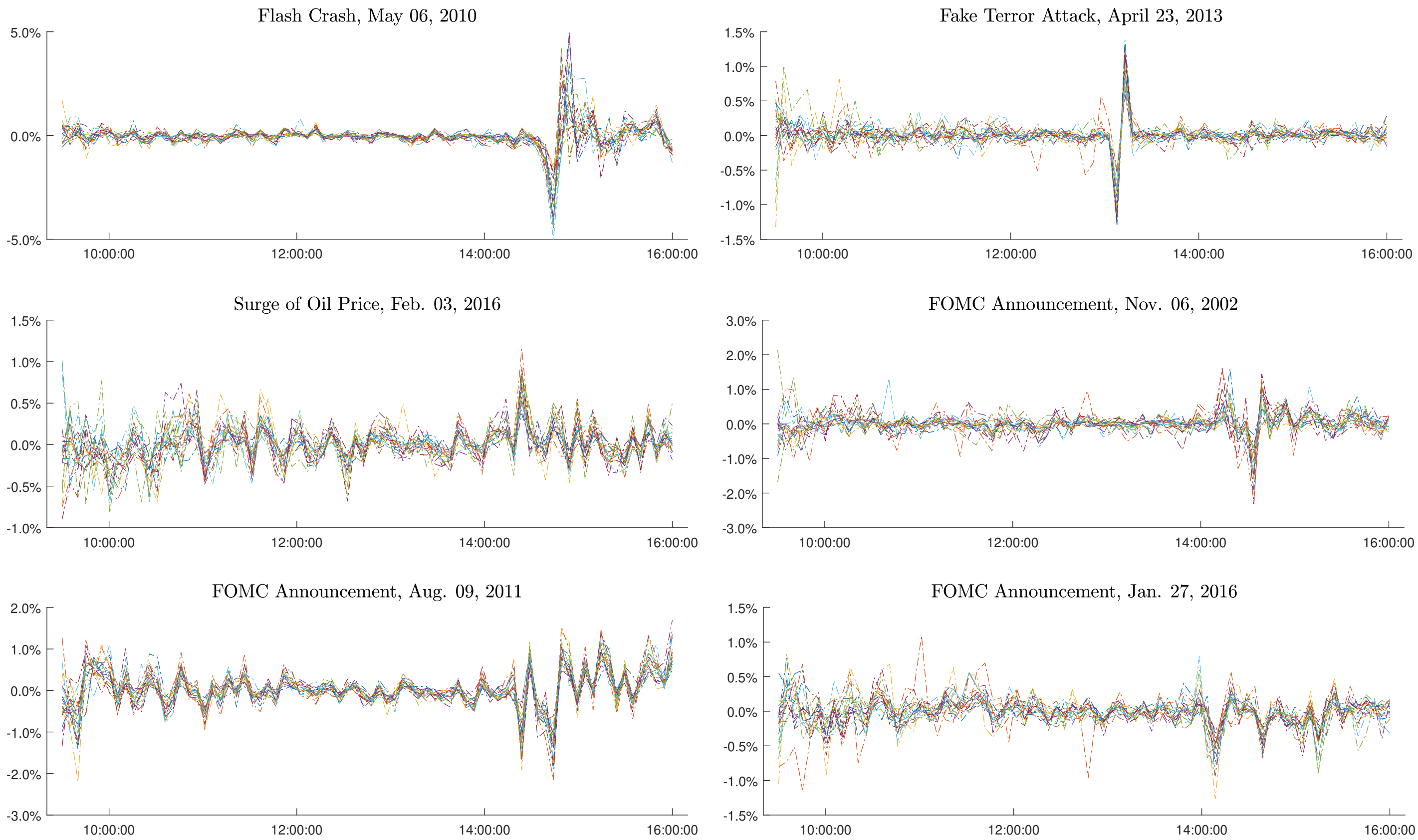
Note: The table reports the out-of-sample forecast loss for the different models, along with the p-value of the CPA test of Giacomini and White (2006) based on a 5% significance level. Bold numbers indicate that the forecast losses are smaller than that of the HAR model, while starred numbers highlight the forecasts whose losses are significant smaller relative to the HAR model.

Table 8: Minimum Variance Portfolios

	Daily Rebalancing		Weekly Rebalancing		Monthly Rebalancing	
	Δ_6	TO	Δ_6	TO	Δ_6	TO
HAR		0.733		0.524		0.302
HARJ	39.810*	0.711	8.131	0.518	7.291	0.297
HARCJ	96.709*	0.669	54.092*	0.490	100.256*	0.254
HARJ†	11.978	0.729	4.895	0.521	3.071	0.301
HARCJ†	74.038*	0.700	23.711*	0.514	50.681*	0.267

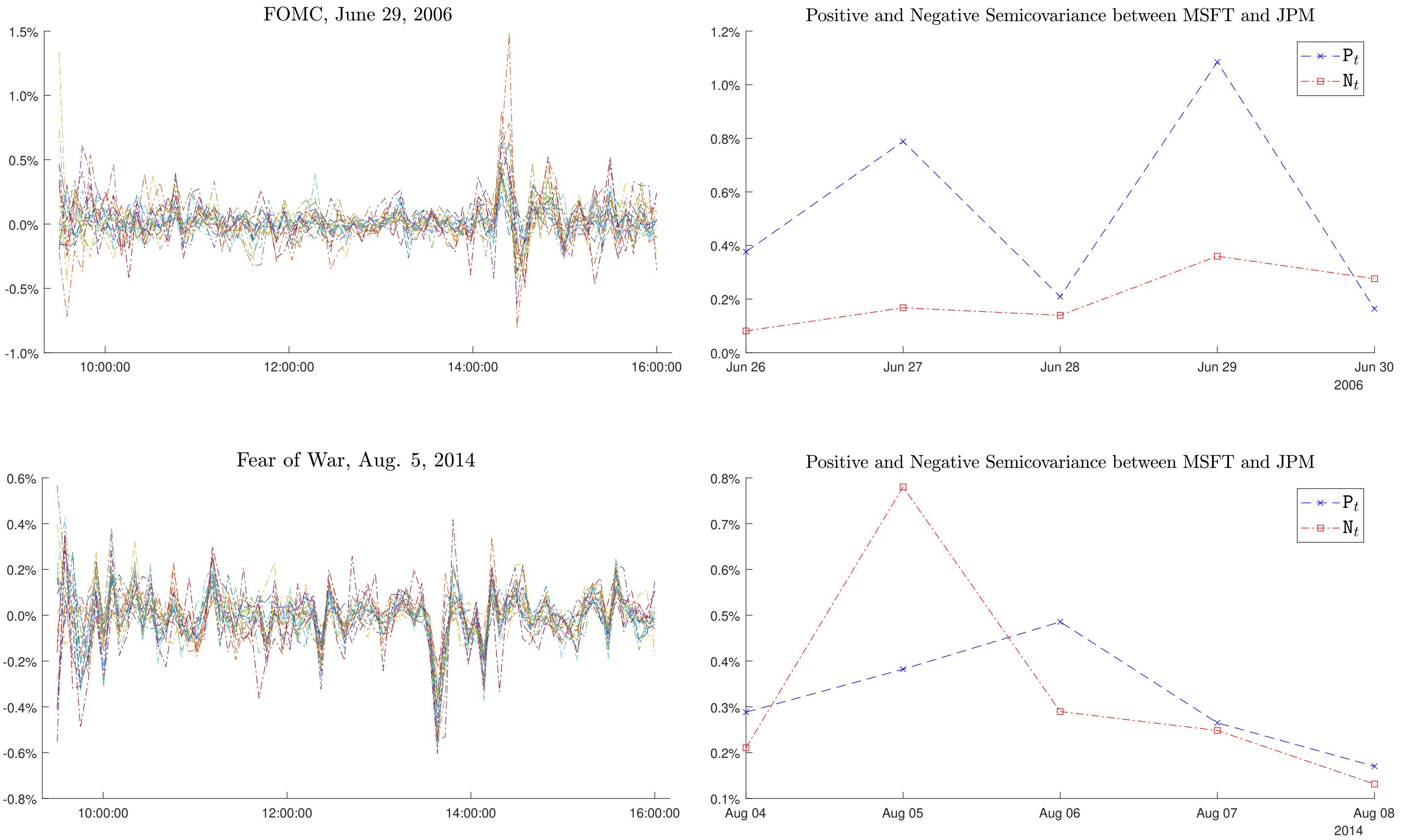
Note: The table reports the results for the global minimum variance (GMV) portfolio using daily, weekly and monthly rebalancing. The results show the average turnover (TO) and the annual basis points (Δ_6) that an investor is willing to sacrifice to switch from the HAR model to one of the models that utilize the common jumps or directional common jumps. The dagger (†) denote the use of directional common jumps, and starred values indicate that Δ_6 is significantly different from zero using a one-sided test. The analysis uses a risk-aversion $\gamma = 6$ and transaction cost $c = 0.5\%$.

Figure 1: Intraday Returns on Days with Simultaneous Jumps



Note: The figure depicts the 5 minutes intraday returns of the 20 individual stocks on 6 different days where simultaneous jumps where identified in our sample. The dates correspond to well known Flash Crashes, FOMC meetings and the surge of the WTI oil price which increases by 8% in a single day.

Figure 2: Semicovariance and Directional Common Jumps



Note: The figure depicts the 5 minutes intraday returns of the 20 individual stocks and the positive and negative realized semicovariances between MSFT and JPM for two different days. The top (bottom) panel shows results based on June 29, 2006 (August 5, 2014).